HEAT TRANSFER IN THE LAMINAR FLOW OF STRUCTURALLY VISCOUS FLUIDS

V. I. Popov and E. M. Khabakhpasheva

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It was shown in [1] that the fluidity of structurally viscous fluids varies linearly in a range of shear stresses of practical interest.

According to [2], the heat transfer coefficients in the laminar flow of such fluids with constant physical properties and with internal heat sources neglected are given by the relationship

$$\mathrm{Nu} = \left[\left(1 + \frac{\Theta}{\varphi_0} \tau_{W} \right) / \left(1 + 0.8 \frac{\Theta}{\varphi_0} \tau_{W} \right) \right]^{1/3} \mathrm{Nu}_{0}.$$
 (1)

We investigated heat transfer with a 4% solution of polyvinyl alcohol (PVA) and a 1.25% solution of carboxymethylcellulose (Na-CMC) in water in order to confirm relationship (1) experimentally. The rheological characteristics of the investigated structurally viscous liquids were determined on a modified Ubbelohde capillary viscometer and are shown in Fig. 1. The figure shows that the fluidity curve $(\varphi - \tau)$ is approximated satisfactorily by a linear law for a 1.25% Na-CMC solution in the whole investigated range of shear stress and for a 4% PVA solution up to $\tau_{\rm W} \approx 4.5$ N/m². The heat transfer experiments were carried out in the same range of shear stress. The ther-



Fig. 1. Curves of fluidity φ (m²/N·sec) for a 4% solution of PVA in water (a) and for a 1.25% solution of Na-CMC in water (b): 1) 70° C; 2) 50; 3) 30; 4) 20; 5) 35; 6) 14; $\tau_{\rm W}$ in N/m².

mal conductivity of the structurally viscous liquids was determined by the method of [3].

The experimental apparatus consisted of a closed cycle with a constant-level tank and a cooler, which ensured the constancy of the flow and temperature of the liquid at the entrance during the experiment.

The experiments were conducted in a rectangular channel ($1 \times x 4 \times 100$ cm) with the heat supplied to one side and with the condition q_w = const. The mean flow velocity of the liquid was determined by a rotameter, which was graduated beforehand on the investigated liquids by means of the optical instrument described in [4].

The operation of the apparatus was tested in special experiments in water and then the values of Nu_0 for $Pe_0 d/x$ numbers in the range 70 to 3000 were obtained in experiments with water-glycerol solutions.

In experiments with a water-glycerol solution of concentration 75% by weight, the viscosity of which was approximately the same as that of the investigated structurally viscous liquids, we determined the limits of the possible changes in the regime parameters (velocity, temperature, heat flux) at which the effect of natural convection could be neglected. The obtained results agreed with the recommend-ations of [5].



Fig. 2. Heat transfer coefficients α (W/m² · deg) for a 4% solution of PVA in water with different heat loads and different flows of liquid: I) < W > = 12.7 cm/sec; II) 6.7; III) 3.1; 1) x/d = 22.4; 2) 35.2; 3) 43.2; q in W/m².

Estimates showed that the effect of dissipation of mechanical energy on the heat transfer in our experiments was also negligible.

As mentioned above, relationship [1] is valid for quasi-isothermal conditions, where the change in the physical properties of the liquids over the cross section of the flow can be neglected. In the experiments, however, even at comparatively low heat loads ($q = 2 \cdot 10^4$ W/m²) the change in viscosity over the cross section of the flow was considerable. To obtain the values of the heat transfer coefficient in quasi-isothermal conditions we carried out the experiments in the following way.



Fig. 3. Plot of $Nu_0\chi^{-1/3}$ against $Pe_0 d/x$ for laminar flow: 1) for water-glycerol solutions of concentrations 75 and 37.5% by weight; 2) for 4% PVA solution; 3) for 1.25% Na-CMC solution.

We measured the heat transfer coefficients at constant rate of flow with several different heat fluxes. By extrapolation of the heat transfer coefficients into the region of zero heat fluxes we reduced the experimental data to quasi-isothermal conditions (Fig. 2). The values of the Nusselt number obtained in this way for structurally viscous liquids and water-glycerol solutions are given in Fig. 3. For comparison with formula (1) the values of the Nusselt number for structurally viscous liquids are multiplied by $\chi^{1/3}$, where

$$\chi = (1 + \Theta/\varphi_0 \tau_W)/(1 + 0.8 \Theta/\varphi_0 \tau_W).$$

The experimental points lie satisfactorily close to a line, thus conforming the validity of formula (1). The maximum deviation of the experimental data from the averaging line does not exceed $\pm 8\%$.

NOTATION

Nu₀ is the Nusselt number for laminar flow of Newtonian liquid; Nu is the Nusselt number for structurally viscous liquids; Pe₀ is the Peclet number determined from temperature at entrance to channel; Re₀ is the Reynolds number determined from zero fluidity $(\tau_w^{\rightarrow 0})$ and temperature of liquid at entrance; $\chi = (1 + \Theta/\phi_0 \tau_w)/(1 + 0.8\Theta/\phi_0/\tau_w)$ is a coefficient which takes into account the structurally viscous properties of a liquid with a linear fluidity law; Θ is the coefficient of structural instability of the liquid; φ_0 is the zero fluidity of liquid; τ_W is the tangential shear stress on wall.

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Institute of Thermophysics, Siberian Division of AS USSR, Novosibirsk

HEAT TRANSFER IN THE RADIATIVE HEATING AND DRYING OF GRANULAR MATERIAL IN A FLUID-IZED BED

A. G. Gorelik

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We consider the case where all the heat supplied to the fluidized bed by radiation through the upper boundary is carried off by the fluidizing medium (gas). The propagation of heat into the bed depends on the effective longitudinal thermal conductivity λ_{eff} . Neglecting longitudinal conductive heat transfer by the gas and regarding the bed as ideally fluidized and consisting of smooth spherical particles of equal diameter we write the Fourier-Kirchhoff equation for unit elementary volume:

$$\lambda_{\text{eff}} \frac{d^2 \Theta}{dh^2} = \frac{6\alpha (1-\varepsilon)}{d} (\Theta - t), \qquad (1)$$

$$\gamma_g c_g w_g \frac{dt}{dh} = \frac{6\alpha (1-\varepsilon)}{d} (\Theta - t).$$
 (2)

Introducing the dimensionless height y = h/H of the bed and using the symbols $a = \lambda_{eff}/H$, $b = 6\alpha(1 - \varepsilon)H/d\gamma_g c_g w_g$, $g = b\gamma_g c_g w_g = 6\alpha(1 - \varepsilon)H/d$, we obtain

$$a\frac{d^2\,\Theta}{dy^2} - g\left(\Theta - t\right) = 0,\tag{3}$$

$$\frac{dt}{dy} - b\left(\Theta - t\right) = 0. \tag{4}$$

 $\mathsf{Expressing} \, \Theta$ from (4) and substituting in (3) we obtain after transformations

$$\frac{d^3t}{dy^3} + b \frac{d^2t}{dy^2} - \frac{g}{\alpha} \frac{dt}{dy} = 0.$$
 (5)

The general form of the solution of Eq. (5) is

$$t = C_1 \exp\left[-\frac{b}{2}(1-\Gamma)y\right] + C_2 \exp\left[-\frac{b}{2}(1+\Gamma)y\right] + C_3, (6)$$

where

$$\Gamma = \sqrt{1 + 4g/ab^2}$$

Using Eq. (4), we obtain an expression for

$$\Theta = C_1 \frac{1+\Gamma}{2} \exp\left[-\frac{b}{2}(1-\Gamma)y\right] + C_2 \frac{1-\Gamma}{2} \exp\left[-\frac{b}{2}(1+\Gamma)y\right] + C_3.$$
(7)

The boundary conditions are obtained from the following considerations. Since the absorption of radiation terminates in a thin upper zone of the bed [1, 2] we can assume that the radiative heat flux is delivered to the upper cross section of the bed; the heat is carried downward by the effective thermal conductivity of the bed. Hence, when y = 1,

$$a \ \frac{d \Theta}{dy} = q_{\rm rad}.$$
 (8)

There is no heat flux through the lower boundary of the bed and, hence, for $\mathbf{y}=\mathbf{0}$

$$a \ \frac{d \Theta}{dy} = 0. \tag{9}$$

In addition, when y = 0, $t = t_1$.

Using the boundary conditions we finally obtain expressions for the gas temperature

$$t = t_{1} + q_{rad} b \exp\left(-\frac{b}{2} y\right) \operatorname{sh}\left(\frac{b}{2} \Gamma y\right) \times \left[g \exp\left(-\frac{b}{2}\right) \operatorname{sh}\left(\frac{b}{2} \Gamma\right)\right]^{-1}$$
(10)

and for the temperature of the material

$$\Theta = t_1 + \left\{ q_{rad} b \exp\left(-\frac{b}{2} y\right) \left[\operatorname{sh}\left(\frac{b}{2} \Gamma y\right) + \Gamma \operatorname{ch}\left(\frac{b}{2} \Gamma y\right) \right] \right\} \times \left[2g \exp\left(-\frac{b}{2}\right) \operatorname{sh}\left(\frac{b}{2} \Gamma\right) \right]^{-1}.$$
 (11)